CONSIDERATION OF TRENDS IN THE OPTIMIZATION OF FARM SYSTEMS UNDER RISK CONDITIONS

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ABSTRACT

The goal of this study was to develop a mathematical programming model which allowed the optimization of production systems in regard to trends in the results of economic activity, with consideration to contingent variations. The formulation of this model is based on the optimization of inter-annual deviations in the historical series of the results of economic activity. Numerical simulations of this model were compared to solutions obtained from the minimization of variance and the minimization of absolute deviation models. Both models were tested with and without the use of linear regressions. The obtained results reveal that the optimized model of inter-annual deviation selected activities by the trends in their economic results, increasing the level of activities with expanding results and decreasing the levels of activities with diminishing results in their solutions. The application of linear regression to the minimization of variance and minimization of absolute deviation models has not enabled these deviations to distinguish trends in the results. In the absence of positive or negative tendencies in the historical series, the optimization model of inter-annual trends revealed a superior capacity to reduce the variance than the minimization of the absolute deviation model.

Key words: Optimization, Production Systems, Risks

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1. Introduction

One of the most important characteristics of the farm and cattle breeding activity is the high risk it is subject to. Thus, all of the mathematical models worked out in the study of production plans for the increase of farm profit must include the risk in their formulation. However, the consideration of the risk in units of production, especially in respect to mathematical programming models, brings problems that have not yet been solved. One of these problems lies in the assumption, used in the majority of these models, that variation in the economic results of farm units is merely aleatory, what is not highly probable. Thus, those models do not separate the aleatory variations in the trends of the economic results of farm units caused by permanent changes in the economy, such as:

-persisting offer expansion, caused by higher productivity or the emergence of new production regions;

-decrease in demand, caused by substitute products on the market;

-changes in economic policies, etc.

Moreover, progressive changes in the yields can occur even with the regular use of technology, such as, for example, profit decreases caused by erosion or depauperation of chemical elements in the soil, or, on the other hand, progressive increases which can occur from the improvement of physical and chemical soil conditions by the application of crop rotation, conservation and soil treatment, etc.

Another implicit assumption found in risk optimization models is that farmers, in making their decisions, are unconcerned about increasing or decreasing profit activity over a period of time. Thus, according to those models, the farmers base their decisions only on the average and variability of the amount of profit obtained from their activities, disregarding the trends of eventual economic results. We believe that these assumptions can lead optimization models to select plans which could be unacceptable to farmers, seriously compromising the effectiveness of mathematical programming in its use as a tool for farm management.

The objective of this study is to present and evaluate a mathematical programming model which will enable us to examine the trends of the economic results of production units, taking adverse variations into appropriate consideration.

2. Theoretical Foundation

2.1. Models used for the Optimization of Farm Systems under Risk Conditions

The programming model proposed by Markowitz (1952), has been widely used as a reference by which to test the effectiveness of new models, especially those using linear programming. In its simplified version, this model can be written as:

$$\text{Minimization } V = \Sigma \Sigma x_j x_k \sigma_{jk}$$

Subject to restrictions

$$\Sigma a_{ij} x_j + \Sigma a_{ik} x_k <= b_i$$

$$\Sigma c_j x_j + \Sigma c_k x_k = E$$

$$x_j, x_k >= 0$$

Where
V = total variance of economic results of the activity;
x_j, x_k = level of activities j and k, respectively;
s_{jk} = covariance between the economic results of activities j and k, when j and k are different; if j = k, s_{jk} corresponds to activity variance;
a_{ij}, a_{ik} = technical requirements of activities j and k in relation to the application or limitation i;
b_i = availability of the application i;
c_j, c_k = economic result of units of activities j and k, respectively;
E = total expected economic result;

Markowitz's model (1952), also known as the expected income-variance model (E-V), considers the variance of the economic results as a risk measure of which is minimized in respect to technical limitations and the expected economic result, which is fixed, "a priori." For each solution the model produces, the value of E is changed, obtaining a minimum of V for each E. The set of these pairs of values constitutes a border E-V.

As this model was originally developed for the optimization of financial applications, the economic result was expressed in terms of relative valorization of the documents (at normal percentages), being that the variance refers to a measure of deviation in relation to this valorization. Thus, the variations due to the valorization of the documents were automatically separated from the random variations in the calculation of variance. When this model started being applied in the optimization of farm systems, the variance started being calculated in relation to an arithmetic average of the economic results of the activities. Therefore, the possible variations of these results originated by systematic increase and decrease of the economic results of farm and cattle breeding activities, principally those originating from tendencies in price variation, were considered as random variations, such as variations in physical productivity due to climate problems. Such confusion can become extremely serious in distinguishing among the options available to the farmers, activities with high tendencies of increasing or decreasing economic results. In that case, the results from current formularizations applied by Markowitz (1952) might result in ineffective plans regarding the risks which the farmers obviously try to avoid.

Several authors have alleged that Markowitz's E-V border model only applies when the economic results follow a normal distribution pattern or when the utility function of the economic agent is quadratic (Robinson & Brake, 1979; Kaylen and others, 1987). In both cases, the distribution of economic results can be expressed in relation to its average and variance. These limitations have instigated the creation of risk models which provide direct solutions for non-linear approximations of the utility function

(Kaylen and others, 1987; Lambert and others, 1987). However, empirical evidence obtained from comparing the results of E-V models to models based on several estimates of the utility function reveal a large similarity among obtained results (Kroll and others, 1984). Even though the measures used to obtain this evidence have been rejected by some authors (Pulley, 1985), they have, on the other hand, been supported by others (Reid & Tew, 1986). Thus, even many decades after its appearance and the debate about its capacity to adequately simulate the behavior of the economic agents, Markowitz's E-V model (1952) continues to be one of the main references used in the development of math programming risk models.

One of the models most widely used as an approximation of Markowitz's model (1952) is the model suggested by Hazell (1971), named MOTAD (Minimization of Total Absolute Deviations) by this author, also known as E-A Border Modeling (Kennedy & Francisco, 1974).

Its formularization, suggested by Hazell (1971) can be written as:

$$\text{Minimization } W = \Sigma y_t\text{-}$$

Subjected to the following restrictions:

$$\Sigma\,(c_{tj} - g_j)\,x_j + y_t\text{-} >= 0$$

$$\Sigma\, a_{ij}\, x_j <= b_i$$

$$\Sigma\, c_j\, x_j = E$$

$$x_j >= 0$$

Where

y_t- = total negative deviation of the farm's economic results in year t;

$c_{tj} - g_j$ = total deviation of the economic result of activity (crop and cattle) in the year t, in regards to the average of the economic results of this activity, where g_j is the average calculated using the economic results from all years and c_{tj}, the economic results of the t-th examination of j-th activity.

x_j = level or amount of activity j units in the solution;
a_{ij} = technical requirements of activity j with the assistance or limitation of i;
b_i = availability of application i;
c_j = pre-established economic result, coefficient of paraversification, rising from zero to the optimal given result for the resolution of the conventional linear programming model to maximize economic results.

Though this model is normally considered a linear approximation E-V model, it can also be interpreted independently of the variance minimization model, even with the absolute deviation interpreted statistically without being transformed into variances (Kennedy & Francis, 1974).

2.2. The Use of Regressions in the Identification of Tendencies in Historical Series

As previously examined, normal data distribution is one of the assumed postulations in the applicability of quadratic E-V border models (Robinson & Brake, 1979; Kaylen and others, 1987). In calculations where data present trends, this condition is not met. Beginning with the application of the regressions, there is a closer approximation of the data in a symmetrical distribution which decreases the apposition risk with the application of the E-V model, such as with the other models which project symmetrical distributions (Stulp, 1977). While a significant regression indicates a tendency towards positive or negative change of the average result of economic activity over time, this average must also be changed, adopting the value calculated by the regression equation for the corresponding year's activity projection as the average. Another procedure which can also be used is the adoption of the data average from the last 2 or 3 years as an estimate of the average result of the economic activity.

The variances and covariances of the economic results are calculated from the remainders of the regressions in the cases where regression is found to be statistically significant. Thus, the average quadrate of the remainders is used as the variance of the economic results of each activity. The covariance between the remainders of the regressions of the two activities is considered as a covariance between the economic results of these activities.

As the application of the regressions change the average values, increasing them in the case of positive trends and decreasing them in the case of negative trends, it can also lead to the favoring of activities with increasing trends and the fall in the level of activities with decreasing trends in their results. Thus, the application of regression to the data of mathematical programming problems can be a useful procedure in defining production plans and in considering trends in price activity or in physical productivity.

2.3. The Optimization of the Inter-Annual Differences Model (OID)

In this section we will describe the structure of a mathematical programming model which allows the distinction between trends and random variations considering that the latter has not been neglected in the preparation of effective farm and breeding production plans, in relation to risk factors, and in doing without the use of regressions.

Basically, as in the majority of previously mentioned models, this model consists of the minimization of a given dispersion measure with respect to technical limitations and the minimum economic result, defined "a priori". The dispersion measure used consists of the negative difference between the economic results within the years along the available data series. However, as opposed to the other models described, the optimization of inter-annual differences does not consist of a simple deviation minimization. Only the negative differences of the economic results of the activities in the current year minus the economic results from the previous year are minimized and these differences, when positive, can adopt any value. Thus, taking into consideration a "T" year series and a production system with "n" activity potential subjected to "m" technical restrictions, the model could be written as:

$$\text{Minimization } \Sigma N_{t-(t-1)} \qquad (t = 1, 2, \ldots, T)$$

Subject to the following restrictions:

$$\Sigma (c_{tj} - c_{(t-1)j}) x_j + N_{t-(t-1)} >= 0 \quad (j = 1, \ldots, T-1)$$
$$\Sigma a_{ij} x_j <= b_i \quad (i = 1, 2, \ldots, m)$$
$$\Sigma_j x_j = E$$
$$x_j >= 0$$

Where

$N_{t-(t-1)}$ = difference between the economic results from year t and year t-1;

c_{tj} = economic results from activity j in year t, per unit of activity;

$c_{(t-1)j}$ = economic results from activity j in year t-1, per unit of activity;

x_j = level or amount of units of activity j

a_{ij} = technical requirements of activity j in the results;

b_i = availability of i assistance;

c_j = average economic result per unit of activity j;

E = expected total economic result average.

Even though the structure of this model is extremely simple, its characteristics distinguish it from the previously mentioned models. Since deviations are not calculated in relation to the average in this model, as in the previously mentioned models, but in relation to the result from the previous year, not only the magnitude of the results are considered for the optimization process, but the order in which they are presented in the studied series. This allows, in this model, successive increases and decreases in the results to be interpreted as trends. Moreover, when these successive increases or decreases do not occur, but only variations around the average values, this model also has the capacity to identify these variations, giving them, however, less importance in relation to negative trends. If the software used allows the processing of the problem without restricting the non-negativity of the variations, it is possible to formulate a version of the model capable of giving priority to the activities with positive trends in relation to the variables with stable values. If the restriction of the non-negativity is maintained, the model will become unable to distinguish the difference between stable activities and activities with positive trends. This last version, which can be considered more "conservative," is the one which will be used in this study.

Another interesting aspect of this model is that the trends (increasing or decreasing) of the economic results must exert a strong influence on the farmers in regard to their production plans, that is, the farmers are probably not indifferent to the tendencies of the results. This indicates that the differences in the economic results in relation to the previous years is one criterion which can be considered plausible in simulating the economical conduct of farmers.

3. Methodology

In this study, a comparative analysis of the following models was achieved: minimization of variance (MV), minimization of absolute deviation (MAD), and optimization of inter-annual deviation (OID). Two versions of the MV and MAD models were utilized, with (MVR) and without (MADR) the correction of the data through linear regression. In the formularization of problems with linear regression correction, the expected economic result of the activities correspond to an extrapolation of the economic result based on the regression equation for a one-year-planning horizon, for the case in which the regression was significant, with 5% probability on the F test. In the case in which the F test did not show 5% probability significance, the expected economic result of the activities is the average result of the series. Moreover, the models with correction by linear regression of the variance and absolute deviation average were calculated according to the value of the economic results obtained from the regression equation of each year of the activities in which the regression obtained 5% probability significance according to the F test. For those activities in which the regression was not found to be 5% significant, the variance and the deviations were calculated based on the original data of the series.

Comparative studies were accomplished through two numerical simulations, one with the objective of testing the application of linear regression as a way in which to consider tendencies (simulation 1) and the other with the objective of testing the minimization capacity of the OID model in dispersing the economic result (simulation 2). In both simulations, the average total economic result was fixed at 20,000 units.

The reason why we chose to use numerical simulations in this study is because it allowed us to easily test the effect of various configurations of the historical series on the results supplied by the models, also allowing an analysis of the consequences of the relative expressions regarding risk factors (objective or restricted function) without the influence of technical limitations present in the farm systems models (relative, for example, to land and manual workers available). Thus, the results obtained from the numerical simulations are independent on the level of the fixed economic result, varying linearly at the same rate.

3.1. Data Series used in the Simulations

3.1.1. Simulation 1: Test of the Application of Linear Regression

The data series used in this simulation were produced in order to represent the economic results of the activities over a 10-year period, with 4 different configurations: data with increasing (C series), decreasing (D series), stable (E series) and irregular (I series) values. These series, their averages, and their absolute deviation averages are described in Table 1. Table 1 reveals the different configurations presented in these series. Table 2 displays data from activities D and C, corrected by linear regression (economic results calculated through regression and remainder equations). Table 3 presents the variance-covariance matrix of the series, calculated from the linear regression remainders. (when they were found to have 5% probability significance on the F test).

It is important to note that the data series were produced in order to clearly show the model's capacity in privileging positive trends and penalizing negative trends in the results, and in distinguishing activities with stable values from activities with irregular values. Thus, it is noted that a D series with decreasing values has very low deviation after it was corrected by the regression, showing less pronounced trends than the C series, with increasing values. In addition, the E series, with stable values, was included in the anticipation that its participation would be relatively high in the results, and the I series, with irregular values, was included in anticipation of low participation in the results.

3.1.2. Simulation 2: Test of the Minimization Capacity of the OID Model in Dispersing Economic Results.

In this simulation, three data series were used in which linear regressions did not obtain 5% probability significance on the F test. These series differ, however, in regards to their averages, variances and absolute deviation averages. Thus, as described in Table 4, the X series presents lower averages and deviations while the Z series presents higher averages and deviations and the Y series presents intermediary averages and deviations in comparison to the others. Thus, the data of the series were adjusted so that the three activities X, Y, and Z would simultaneously figure in the results of at least one of the models tested. Table 5 displays the variances and covariances of the variables X, Y, and Z.

Table 1- Data series used in simulation 1, with averages, variances, and deviation averages.

Year	C*	D*	I n.s	E n.s
1	110	270	130	200
2	130	250	260	190
3	140	235	140	200
4	190	220	170	210
5	200	205	320	200
6	210	195	100	190
7	240	180	250	200
8	230	160	130	210
9	260	145	300	200
10	290	140	200	200
Average	200	200	200	200
Variance	3100	1770	5480	40
Absolute Deviation Average	46	36	66	4

Note: * significant linear regression with 5% probability
n.s. non-significant linear regression at 5% probability

Graph 1. Comparison of data series

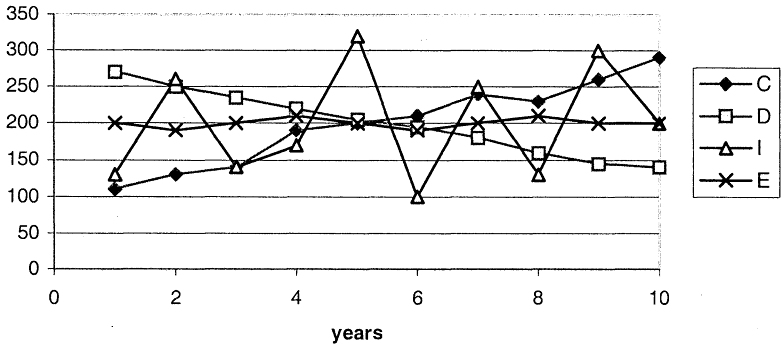

Table 2. Data series used in simulation 1 corrected by regression

Year	C average	C remainders	D average	D remainders
1	114.36	-4.36	265.73	4.27
2	133.39	-3.39	251.12	-1.12
3	152.42	-12.42	236.55	-1.52
4	171.45	18.55	221.91	-1.91
5	190.48	9.52	220.73	-2.30
6	209.51	0.48	192.70	2.30
7	228.55	11.45	178.09	1.91
8	247.58	-17.58	163.48	-3.48
9	266.61	-6.61	148.88	3.88
10	285.64	4.36	134.27	5.73
11	304.67 *		119.67 *	
Abs. Dev. aver.		8.87		2.84

* Note: values estimated through the regression equation

Table 3- Variances and covariances of data series, in regard to their trends

	C	D	I	E
C	112.24	8.15	252.67	3.88
D		9.97	-77.33	-6.58
I			5480	-60
E				40

Table 4- Data series used in simulation 2, with averages, variances and deviation averages.

Year	X	Y	Z
1	131	211	313
2	74	256	135
3	113	27	309
4	89	287	237
5	58	40	0
6	89	172	69
7	58	194	132
8	55	225	390
9	127	132	169
10	133	207	333
Average	92.7	175.1	208.7
Variance	888.61	66,627.29	14,602.21
Abs. average deviation	26.64	65.88	107.7

Table 5- Variances and covariances de X, Y, and Z.

	X	Y	Z
X	888.61	- 158.87	1453.71
Y		6627.29	2799.63
Z			14602.21

4. Results and Discussion

4.1. Simulation 1: Test of the Application of Linear Regressions

The results of the simulation of the minimization of variance and absolute deviation models, with and without the consideration of trends through linear regression, are given in Table 6.

It can be observed from Table 6 that activity C, with increasing values, diminished when data was corrected through linear regression, having been eliminated from the optimal base in the case of the minimization model of absolute deviation averages. In regards to activity D, with decreasing results, its participation pronouncedly increased in the results of both models when linear regression was applied. Such results can be explained through the fact that, even though the application of linear regression had been causing an increase in the values of the economic

Table 6- Comparison of minimization of variance (MV) and minimization of absolute deviation (MAD) models with (R) and without (S) correction of the data by linear regression

	C	D	I	E
MVS	23.16	32.51	0.41	43.93
MVR	12.07	81.72	1.07	31.65
MDAS	18.19	27.27	0	54.55
MDAR	0	90.94	1.74	43.85

results of activity C and a decrease in the values of activity D, the deviations of the latter activity was also pronouncedly diminished. Thus, the decrease in the deviations of activity D had probably exerted more influence on the solution than on the alteration of average values (activities C and D). These results reveal that, in the case of the application of dispersion minimization models in regards to the average (variance or absolute deviation), the activities with sharply decreasing economic results can enter into the solution when the deviations are strongly reduced by linear regression. In this case, these activities can be favored in the solution as opposed to activities with constant or increasing results, contradicting the objectives of considering optimization trends.

Table 7 presents the results of the MVR, MADR, and OID models.

Table 7- Results of the MVR, MADR, and OID models

	C	D	I	E
MVR	12.07	81.72	1.07	31.65
MDAR	0	90.94	1.74	43.85
ODI	50	0	0	50

It can be observed from this table that the exhibited results of the OID model demonstrated only activity C (increasing values) and activity E (stable values), both with 50 units. This solution is very different from the solutions exhibited in the other models, principally in respect to the increased presence of activity C in the results of the OID model and in the presence of activity D in the results of the other models. Activity I was present at lower values in the results of the MVR and MADR models, not appearing in the solution of the OID model. This data indicates that the OID model demonstrated a good capacity for discriminating the revealed trends in the activities, in contrast to those models in which data was corrected by linear regression. One possible explanation for this is that in the minimization of variance and minimization of absolute deviation models, trends are detected just by modifying coefficient values. In the OID model, trends are detected through the actual mathematical structure of the model.

4.2. Simulation 2: Test of the Capacity of the OID Model in Deviation Minimization

Table 8 displays the results of the MV, MAD and OID models regarding the data used in simulation 2. This table also displays the variance and absolute deviation average of the results.

Table 8- Level of activity, variance, deviation standard and absolute deviation average of the results of the MV, MAD, and OID models.

	X	Y	Z	Variance	Standard Deviation	Absolute Deviation Average
MV	143.74	38.12	0	26,250.350	5.123.51	4.260.05
MDA	113.85	38.29	13.14	29,533,740	5,434.50	4,103.25
ODI	163.24	27.80	0	27,358,500	5,230.54	4,324.62

As it can be noted from Table 8 above, the results of the OID model presented a standard deviation (or variance) which is lower than the one presented in the results of the MAD model, having selected the same activity as in the MV model. In relation to the absolute deviation average, results of the OID model presented a value higher than those produced by the other tested models. These results reveal that, in general, the OID model tended to present results closer to the results of the MV model than those presented by the MAD model. Moreover, the results shown in Table 10 indicate that the OID model, in this case, can not be considered inferior to the MAD model in the minimization of the dispersion of the economic result in relation to the average, whereas the results of this estimate depends on the adopted measure of dispersion.

5. Conclusions

The results obtained in this study permit the following conclusions:

1) The transformation of the data by linear regression can not be sufficient to enable the minimization of variance (MVR) and the minimization of absolute deviation models (MADR) to properly discriminate the activities with increasing economic results in relation to activities with decreasing economic results, and to give privilege to the first ones in detriment of the second ones in the results.

2) The optimization of the interannual deviations model (OID) revealed a high capacity for discriminating trends in a historical series, giving privilege to activities resulting in positive trends and avoiding activities resulting in negative trends in the results.

3) In the absence of a data series which show trends, the OID model presented a good capacity for dispersion minimization.

Thus, although the results from this study are preliminary, they indicate that the use of optimization of interannual difference models in studies of production systems can present advantages, especially when trends in the economic results of the activities were detected. However, the results obtained in this study do not invalidate the application of linear regression to data which will be used in the minimization of variance and in the minimization of absolute deviation models. On the contrary, we think that the application of regressions can substantially improve the accuracy of the results obtained with these models. However, it is relevant to point out that the use of models which minimize economic results (variance or absolute deviation) in relation to the average, implies the assumption of neutrality of the farmers in relation to the trends of economic results.

Finally, it is important to stress that, in order to optimize interannual differences, the statistical evaluation of trends in resulting activity must be supplemented with information about real causes of these trends so that extrapolations based on the data series can be done from a more secure basis.

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